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Trapping Atoms by the Vacuum Field in a Cavity.

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Abstract. – We analyse the force on an atom resonantly coupled to the vacuum field in a high-Q microwave cavity. The atom is prepared in the upper level of the atomic transition coupled to the cavity mode. The force results from the reversible exchange of a single excitation between the atom and the cavity. It derives from a potential which reflects the spatial variations of the r.m.s. vacuum field inside the cavity. Depending upon the atom-cavity dressed state into which the system is prepared, this potential presents either a maximum or a minimum at the point where vacuum fluctuations are largest. Very slow circular Rydberg atoms may be reflected back as they enter the cavity, or trapped around cavity centre.

Atoms are subjected to forces in the vicinity of metallic surfaces. A ground-state atom, at small distance to surface d, experiences a $1/d^3$ van der Waals interaction [1] and at large distance a retarded $1/d^4$ Casimir-Polder potential [2] (small and large d are defined by comparison to $\lambda/2\pi$, where λ is a typical atom absorption wavelength). These interactions are always attractive to the surface. It may be less recognized that excited-state atom-metallic surface potentials are, for large d values, of a different nature: they exhibit spatial oscillations *vs.* d, corresponding to sign-changing forces able to pull the atom towards points located away from the surface, at distances of the order of the typical atomic emission wavelengths [3]. This interaction exists in the absence of any applied electromagnetic field and can be viewed as a dispersive effect related to the modification of the vacuum field in the vicinity of the metallic boundary. The depths of the corresponding potential wells are very weak, typically an order of magnitude smaller than the excited-state line width Γ. The excited-atom boundary force is however expected to be enhanced when the atom is inside a cavity structure, resonant or quasi-resonant with an atomic emission frequency from the excited state of interest [4]. Atom and cavity are then conveniently described as a closely coupled entity, the «dressed atom-cavity system» [5]. This concept turns out to be very useful to describe all cavity QED effects where atom and vacuum field are coupled in a

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strong, intrinsically nonlinear way. In this letter we present a very simple computation of the atom-cavity force in the dressed atom-cavity picture. We show that this force depends upon the spatial variation of the vacuum field mode resonant with the atomic transition in the cavity. Depending upon the way the excited atom is introduced in the cavity the force is either repulsive, inducing the atom to «bounce out» of the cavity, or attractive towards the cavity centre, allowing one to envision «vacuum force atomic traps». Orders-of-magnitude estimates show that these effects, although very difficult to demonstrate, should in principle be observable with circular Rydberg atoms and superconducting microwave cavities.

The dressed-atom point of view has already been amply used, in the context of cavity QED [5] as well as in atom electromagnetic forces studies [6, 7]. In the former case, the attention is focused on the lowest-energy states of the system, corresponding to zero or very small photon numbers in the field. In light-forces studies, on the other hand, one is primarily interested in very excited states of the system, for which the field is intense and the forces are large. Our aim here is to show that even when the photon number is as small as zero (cavity vacuum), the atom experiences forces related to atom-field excitation exchange. The following analysis is otherwise very close to the one developed in earlier light-forces studies [6, 7].

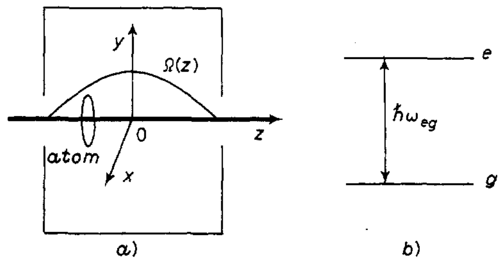

Fig. 1. – *a*) Sketch of atom-cavity system. The atom moves along the $0z$ axis of the cylindrical cavity, with its circular Rydberg orbit in the $0xy$ plane. The sine-wave profile of the cavity mode along $0z$ is shown. *b*) Relevant atomic energy levels. The mode at frequency ω is quasi-resonant with the transition $e \rightarrow g$. Atom enters cavity at level e.

Consider (fig. 1) an atom with two electronic states $|e\rangle$ and $|g\rangle$ separated by the energy interval $\hbar\omega_{eg}$, placed at the point $\boldsymbol{r}(x, y, z)$ in a cavity having an infinite Q mode with angular frequency ω, resonant or nearly resonant with ω_{eg}. In the two-level (TL), single-mode (SM) and rotating-wave (RW) approximations, the atom-cavity system Hamiltonian reads

$$H = \hbar\omega_{eg} D_3 + \hbar\omega(a^\dagger a + 1/2) + \hbar\Omega(\boldsymbol{r})[aD_+ + a^\dagger D_-]. \tag{1}$$

a and $a^\dagger$ are the photon annihilation and creation operators in the mode, $\hbar$ is Planck's constant and D_3 and $D_\pm$ are atomic operators defined as

$$D_3 = \frac{1}{2}[|e\rangle\langle e| - |g\rangle\langle g|], \qquad D_+ = |e\rangle\langle g|, \qquad D_- = |g\rangle\langle e|.$$

The first two terms in eq. (1) are the atom and field Hamiltonians. The last term describes the atom-field interaction. The coupling parameter $\Omega(\boldsymbol{r})$ depends upon $\boldsymbol{r}$ through the spatial distribution $f(\boldsymbol{r})$ of the cavity field, proportional to the mode field amplitude projected along the atomic transition polarization. The function $f(\boldsymbol{r})$ is normalized to unity at maximum. Introducing the effective mode volume $\mathcal{V} = \int |f(\boldsymbol{r})|^2 \, \mathrm{d}\boldsymbol{r}$ and the maximum r.m.s. amplitude of the cavity vacuum field [5] $\mathcal{E}_0 = (\hbar\omega/2\varepsilon_0 \mathcal{V})^{1/2}$, we get the atom-field coupling

$$\Omega(\boldsymbol{r}) = f(\boldsymbol{r})\ \mathcal{E}_0\ d/\hbar, \tag{2}$$

where d is the electric-dipole matrix element between e and g. To be specific, let us consider the case where $|e\rangle$ and $|g\rangle$ are adjacent circular Rydberg states [8] with principal quantum numbers $n = 51$ and 50 and the field is in the TE_{121} mode sustained by a 1 cm-long, 1 cm-diameter cylindrical cavity (1) resonant with the transition frequency $\omega_{eg}/2\pi \simeq 51$ GHz. We then have $\mathcal{V} = 95\ \text{mm}^3$ and $\mathcal{E}_0 = 4.3 \cdot 10^{-3}$ V/m. The atoms propagate along the cavity axis $0z$ and the circular Rydberg states have their valence electron orbiting in the $0xy$ plane (fig. 1*a*)). The $e \to g$ transition is circularly polarized with a large matrix element $d = 10^{-26}$ C.m. The cavity field mode on the $0z$-axis is linearly polarized along $0x$. Half the vacuum fluctuation intensity is thus coupled to the $e \to g$ transition. The variation of Ω along the cavity axis is shown in fig. 1*a*). It exhibits the shape of a sine-wave arch vanishing at cavity ends. The maximum Ω value, at the cavity centre, is $\Omega(\mathbf{0}) = 4.2 \cdot 10^5\ \text{s}^{-1}$.

The eigenstates and energies of H are straightforward to compute [5]. The ground state of the atom-cavity field system is $|g; 0\rangle$, direct product of the atom lower state and vacuum field state. There is no atom-field coupling effect in this state, at least within the TL and RW approximations. The two first excited states of the combined system, called $|+\rangle$ and $|-\rangle$, are linear superpositions of the states $|e; 0\rangle$ (excited atom in vacuum field) and $|g; 1\rangle$ (atom in the lower state with one photon in the field):

$$\begin{cases} |+\rangle = \cos\theta_0 |e; 0\rangle + \sin\theta_0 |g; 1\rangle\,, \\ |-\rangle = -\sin\theta_0 |e; 0\rangle + \cos\theta_0 |g; 1\rangle \end{cases} \tag{3}$$

with

$$\operatorname{tg}(2\theta_0) = \frac{-2\Omega(\mathbf{r})}{\delta} \qquad (0 < \theta_0 < \pi/2)\,. \tag{4}$$

$\delta = \omega - \omega_{eg}$ is the (small) atom-field detuning. The energies of the three lowest atom-cavity dressed states are

$$E_{g,0} = \hbar\delta/2\,; \qquad E_\pm = \hbar\omega \pm \frac{\hbar}{2}\sqrt{4\Omega^2(\mathbf{r}) + \delta^2}\,. \tag{5}$$

The eigenstates and energies depend, through $\Omega(\mathbf{r})$, upon the atom position in the cavity. We have plotted in fig. 2 the dressed energies as a function of the atom location along the cavity axis, in the case of a negative or a positive detuning δ much smaller than the maximum coupling $\Omega(\mathbf{0})$. Equation (5) shows that at cavity ends, where Ω vanishes, the coupling angle θ_0 tends towards 0 if δ is negative and towards $\pi/2$ if δ is positive. As a result, the $|+\rangle$ state becomes then $|e; 0\rangle$ (for $\delta < 0$) and $|g; 1\rangle$ (for $\delta > 0$). Conversely, the $|-\rangle$ state turns into $|g; 1\rangle$ ($\delta < 0$) or into $|e; 0\rangle$ ($\delta > 0$).

We now make use of these energy plots to analyse the motion of an atom coupled to the cavity vacuum. Consider an atom prepared in $|e\rangle$ outside the cavity and moving along $0z$ from left to right, the detuning δ being slightly negative. The atom and cavity are then initially combined in the $|e; 0\rangle$ state, which, at the cavity end, is identical to the $|+\rangle$ eigenstate (see fig. 2*a*)). Provided the atom is slow enough, the system adiabatically follows the upper dressed level and thus gains internal energy, at the expense of its kinetic energy. The positive energy $E_+(z)$ appears as a potential-energy barrier for the atom external degrees of freedom. If the initial kinetic energy is smaller than $\hbar\Omega(\mathbf{0})$, the atom will bounce on the

(1) The TE_{121} mode is often used in Rydberg atom-cavity experiments (see ref. [13]). For a description of the mode geometry, see ref. [9].

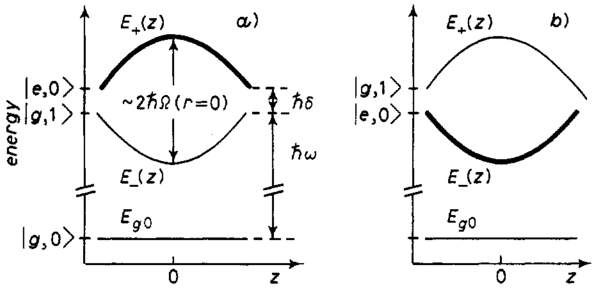

Fig. 2. – Dressed atom-cavity energy levels E_{g0} and $E_\pm(z)$ as a function of atom distance to cavity centre along cavity axis. Atom and field mode are detuned by $|\delta| \ll \Omega(\boldsymbol{r}=0)$. Negative and positive δ situations are shown in *a*) and *b*), respectively. The expressions of the dressed states at cavity ends are indicated in each case ($E_+ - E_-$ is equal to $2\hbar|\delta|$ at both ends). For $\delta < 0$ ($\delta > 0$), the system branches into the $|+\rangle$ ($|-\rangle$) state when the atom enters the cavity. The energy branch describing the potential experienced by the atom in each case is represented as a bold solid line. For clarity, the frequencies ω, Ω and δ are not shown to scale.

cavity vacuum and will be reflected back. If, on the other hand, δ is slightly positive (fig. 2*b*)), the system is initially prepared in the state $|e;0\rangle$, identical to the $|-\rangle$ dressed state, which it subsequently adiabatically follows. The motion of the atom is then determined by the attractive potential well $E_-(z)$, minimum at the cavity centre. The atom is first accelerated, then decelerated as it leaves the cavity on the right. Energy conservation obviously prevents the atom from being trapped in the cavity. Trapping becomes however possible if the atom-field coupling is modified during the time the atom is inside the cavity. One way of achieving it is to tune δ from a large positive value to a very small value as the atom travels towards the cavity centre. In this way an atom with a very small initial velocity could be trapped by the vacuum field in the centre of the cavity (2).

Note that there is no force on an atom introduced in an empty cavity in the lower state g, since E_{g0} does not depend upon $\boldsymbol{r}$ (at least in the limit of the approximations considered here). The force on an excited atom exists because the system evolves in a linear superposition of states $|e;0\rangle$ and $|g;1\rangle$, corresponding to a reversible exchange of excitation between the atom and the initially empty cavity.

We now proceed to estimate the order of magnitude of these cavity-vacuum mechanical effects and discuss the validity of the approximations made in the above analysis. Note first that a coupling $\Omega = 4.2 \cdot 10^5\,\text{s}^{-1}$ corresponds to an energy of $4.4 \cdot 10^{-29}\,\text{J}$ and a temperature of $3.2\,\mu\text{K}$, associated to a mean atomic velocity ≈ 23 cm/s for hydrogen, 5 cm/s for Na and only 2 cm/s for Cs. Although they are very small, temperature and velocities of this order of magnitude have actually been achieved in optical molasses [10]. They correspond to atomic de Broglie wavelengths in the μm range, still far smaller than the cavity size. We are thus justified to describe classically the motion of the atom centre of mass. Note also that at such velocities, the characteristic cavity crossing time T is 0.05 to 0.5 s, long enough for the adiabatic approximation to be valid, provided δ is larger than a few kHz (for $\delta = 10$ kHz, the

(2) For the sake of simplicity, we have considered the TE_{121} mode variation and the corresponding cavity forces *along cavity axis*. Three-dimension generalization is easy. The mode is linearly polarized parallel to the $0xy$ plane, with a spatially varying direction (see ref. [9]). In a $0xy$ plane, $\Omega(\boldsymbol{r})$ exhibits radial oscillations described by Bessel functions with a relative maximum on axis, the absolute maximum being at $\boldsymbol{r}=0$. As a result, the vacuum force corresponds, for positive δ, to a *three-dimensional stable trap* attracting the atom towards the cavity centre.

nonadiabatic transfer rate during cavity crossing is smaller than 10^{-4}). Another important consideration is the lifetime of the atom-cavity dressed state, compared to T. The system excitation can decay by two channels, either because the atom spontaneously decays to a lower state, or because the cavity relaxes back to vacuum (photon dissipation). The circular Rydberg states with $n = 50$ or 51 have a natural decay rate $\Gamma \approx 30\ \mathrm{s}^{-1}$ in free space. In an ideal infinite Q cavity, however, this irreversible decay is totally inhibited since all the free space modes are suppressed and the atom-cavity excitation would survive indefinitely. A realistic superconducting cavity at subKelvin temperatures can have a photon damping time τ_{cav} in the second range [11]. The excitation of the atom-cavity system can thus be realistically maintained during a time of the order of several T, making the bouncing or the trapping effects observable. We next consider the validity of the TL, SM and RW approximations. The contributions to the atom-cavity energy shifts due to all the field modes and atomic energy levels neglected in the above analysis depend in a complicated way on the actual cavity geometry. In the vicinity of the cavity centre, we expect these contributions to be at most of the order of the Rydberg atom–single-surface interaction at a distance of 0.5 cm, about $1\ \mathrm{s}^{-1}$ (in angular frequency units). This is 5 orders of magnitude smaller than the resonant interaction, and can thus be neglected. Qualitative estimates of the effect of the atom-cavity coupling on the atomic states also show that the circular electron orbits should remain stable in shape and direction, with negligible contamination by noncircular Rydberg sublevels belonging to the energy manifolds of states e and g ([3]). At last, let us compare the atom-cavity vacuum interaction with the gravitational atom-earth potential. The maximum trap depth $\hbar\Omega(0)$ corresponds for hydrogen to a drop of 2.7 mm in the earth field (only 20 μm for Cs). These lengths have to be compared to the distance from cavity centre to walls (about 5 mm). It thus appears that an atom-cavity vacuum trap would require a somewhat reduced gravity environment for hydrogen and practically zero gravity for heavy atoms.

It is interesting to compare the cavity-vacuum force to the one produced by an excited cavity mode, in the microwave or optical domains. The trapping of atoms in an excited microwave cavity has been proposed as a way of confining cold hydrogen atoms [7]. If the cavity contains N photons, the atom-cavity levels to consider are linear superpositions of $|e;N\rangle$ and $|g;N+1\rangle$ and the atom-field coupling becomes $\Omega(\mathbf{r})\sqrt{N+1}$, much larger than in the cavity-vacuum case ($N = 0$). For example 10^6 microwave photons, still a very weak field, will produce for Rydberg atoms a 3 mK deep trap. Note that much larger fields are required for hydrogen ground-state trapping, because the relevant atomic transition is much weaker. Of course, the same concepts apply in the optical domain, and the dressed-atom analysis has already been extensively used to explain stimulated light forces, optical channelling and related effects [6]. Let us stress two differences between cavity-vacuum–induced and light-induced forces on atoms: i) in usual atom light forces, spontaneous emission is important. It induces transitions between dressed states corresponding to decreasing dressing photon number N, as the light field photons are continuously scattered into empty «reservoir modes». The system randomly jumps between dressed states corresponding to both signs of the force, resulting in a dissipative force with large fluctuations. As noticed in ref. [7] the situation is different in a microwave force experiment where quite generally spontaneous emission is very weak. In a closed infinite Q cavity, spontaneous emission in «reservoir modes» is suppressed altogether and there are no force fluctuations at all. The system is conservative. In a closed cavity with a finite high Q, dissipation affects primarily the field

([3]) We assume that there are no static electric or magnetic fields in the cavity. Static field components in the $0xy$ plane would affect the stability of the circular Rydberg orbits and considerably complicate the present analysis. In a real experiment, small stray fields are unavoidable, but their effect could be suppressed by maintaining a small «directing field» on the atom (see ref. [12]).

and not the atom. When the photon vanishes, the system jumps into the $|g; 0\rangle$ state. This results in a sudden switching-off at a random time of the otherwise conservative vacuum force. ii) Usually one considers the effect of light forces on ground-state atoms quasi-resonantly coupled to an upper level. The cavity-vacuum force, on the other hand, affects excited states coupled to a lower state by the cavity mode. As a result, the intrinsic sign of the force relative to the sign of the atom-field detuning is changed: optical forces pull the atom towards strong fields when δ is negative and towards weak fields when δ is positive. On the contrary, the cavity-vacuum force pulls the atom towards strong vacuum fluctuations when δ is positive, away from them when δ is negative.

Cavity QED forces similar to the one analysed here would have a significant effect on the operation of a micromaser [13] using a beam of very slow circular Rydberg atoms. As the field builds up in the cavity, the atom-cavity exchange force should increase and perturb more and more the motion of successive atoms crossing the cavity. The atom-field interaction time in particular should depend upon the cavity photon number N through this kinetic effect. As a result the field statistics, which strongly depends upon this interaction time, should also be affected. In this way, a subtle connection between the quantum manipulation of the external and internal degrees of freedom of the atom-field system could be established.

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